Spectral Analysis of EUR/USD Currency Rate Fluctuation Based on Maximum Entropy Method.

Present work continues the cycle of articles dedicated to the new Adaptive Trend & Cycles Following Method, based on the state-of-the-art digital technologies for data processing. The first description of the AT&CF-method was presented in the December 2000 issue of the "VALYUTNY SPECULYANT" (CURRENCY SPECULATOR) magazine. In correspondence with [1] main task to be preliminarily solved during the trading algorithm and its adaptation to the specified market development is the spectral estimate of the power (SDP) of the market prices deviations. In particular case for FOREX – it is making spectral analysis of the rates variations of different currency pairs. Present publication is devoted to this specific problem solution that for the first time represents possessing high spectral resolution the S_f estimate of SDP currency rate variations for EUR/USD calculated according to the maximum entropy method. The work gives rationale to the necessity of using parametric methods of spectral estimate for computation the SDP currency rate variations. In one of the following issues of the 'VALYUTNY SPEKULYANT" magazine algorithm based on AT&CF method will be published that allows to generate trading signals for speculating dealership on the EUR/USD market. Besides the exchange charts will indicate all entry and exit market points beginning with January 1999. It allows the reader being interested in to gain in details an understanding the reasons of so high effectiveness of the new AT&CF method. A special attention will be given to studying the timing P&L (profit – loss) performances of the trading system.

Choice of Spectral Analysis Method

Success or failure of the trading algorithm worked out on the base of Adaptive Tendency & Market Cycles Following Method is by 50 % defined by the quality of the SDP estimator. And this is quite natural. In order to use market cycles in future trading algorithm in some way it is necessary to find out beforehand what harmonic components (amplitude and oscillation period is meant) are in the input signal spectrum and then to investigate their properties. It is obvious that this task should be solved with the help of spectral or harmonic analysis. But what method indeed should be chosen for getting consistent SDP estimate with fairly high spectral resolution including? The answer to this question is nontrivial. The readers wishing to liberalize (enlarge their horizon) in this area I recommend to address the splendid survey on spectral estimation [2]. Contemporary methods of spectral analysis include two main classes or categories, namely: parametric and non-parametric methods. Among the category of the parametric methods of the spectral analysis are those methods that establish some set pattern of the spectral density and the task of estimating the pattern parameter based on the results of the appropriate process monitoring on the time bound period. Original (base) model can have the most various views. Spectral density of the time series as a rational function can be used as such model. In this meaning we can distinguish autoregressive pattern (AR) that is corresponded with rational function without zero, pattern of moving averaging-out that is corresponded with rational function without poles, and the pattern of autoregressive moving average that is corresponded with rational function of the most general view with zero and poles. Accordingly, methodologically different approaches are possible for parameters estimation of such rational patterns. Alternatively some variation principle and some functional quality estimation can be chosen as another pattern variant. In this case Lagrange coefficients will act as estimated parameters. Spectral density on maximum entropy method is estimated exactly in this way, where it is required to maximize the process of entropy with some known values of correlation function. Non-parametric methods for spectral estimate differ from parametric ones in absence of some preliminarily set patterns (models) in developing the tasks of spectral estimate. In this class for the spectral density estimate of the set time series many various methods exist. One of the most commonly used method is that at the initial stage the process periodogram that is squared module Fourier transformation of the given realization or some of its modification is computed. After that the task comes to the corresponding window choice that must meet some contradicting requirements. Another widely known and used Blackman and Tukey method provides that for the observed historical series Fourier transformations of the window estimate of correlation sequence are found. At last another approach lies in leading the problem of estimation the spectral density of the historical series to the solution of fundamental integral equation describing Fourier transformations of observed time series through random process with orthogonal increments. From above said follows that the task of spectral estimate does not have unique solution. The choice of the relevant procedure either it is parametric or non-parametric is defined exclusively by the character of the task under solution. In particular it is necessary to consider such factors as availability or absence a priori information on physical characteristics of the investigated process, the possibility of preliminary testing various parametric and non-parametric methods, the time of computing, required memory and so on an so forth. I believe that it is impossible to achieve a qualitative assessment of the SDP currency rate variations using classical non-parametric methods of spectral estimate based on calculation the discrete Fourier transform (DFT) the time series. The reason is hidden in non-stationary state of the currency rate deviations with moving average values almost always depending on the time of their calculation. Strictly speaking, the notions “spectrum” and “spectral density” a priori mean stationary state of the processes they are calculated for.

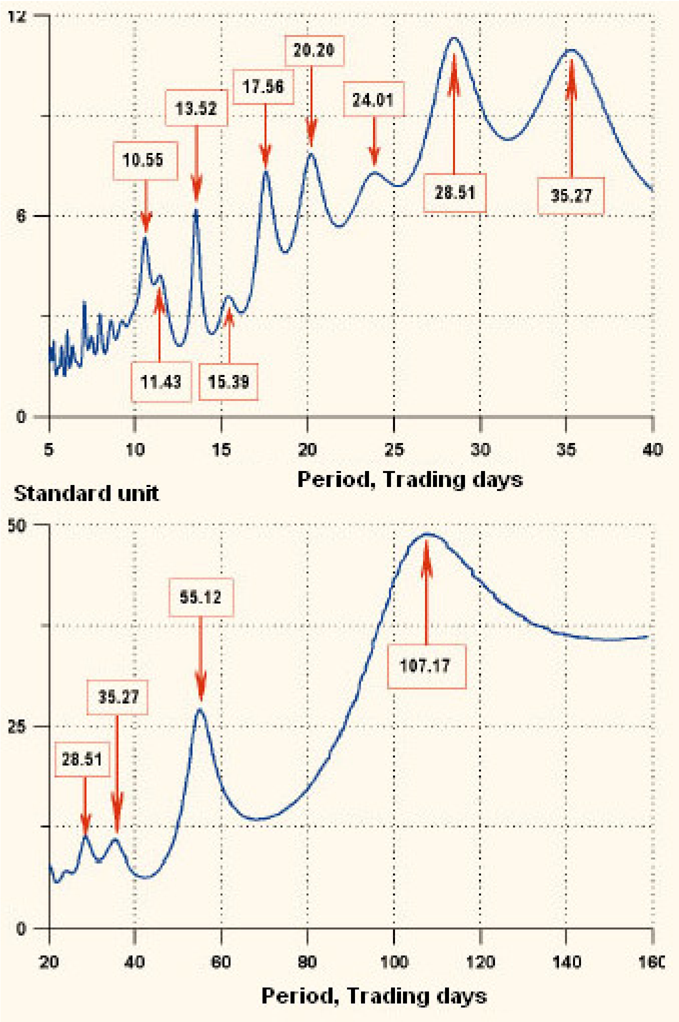

Fig. 1. Spectral density of the power of EUR/USD currency rate calculated with method of maximum entropy. SDP pattern used in calculation is equivalent to autoregressive pattern of 150 order

Attempts to use classical Fourier methods for SDP estimate certainly non-stationary process can only lead to definition the general form of spectral density with amplitude proportionate 1/f, where f is a normalized frequency. With all this including, essential for us details (spectral peculiarities) will turn out to be diffused. In the work [3], for example, it is stated that parameter values are similar and equal to 0.618 for such currency rates as EUR/USD, USD/CHF, GBP/USD and USD/JPY. Why does SDP estimate of non-stationary currency rate calculated with algorithm of periodogram method turn out to be unfounded? The answer to this question is very easy. Firstly, resolving capacity of df spectral analysis and observance interval are connected with plain dependence [4]: $F=K_0/df$, where K_0 – is a coefficient defined by the view of the window function. From this a conclusion follows: the more observance interval E of separate selection is, the higher df spectral resolution is. Secondly, for reducing dispersion SDP estimate it is necessary to make the result average over sufficiently bigger number of selections N (usually $N > 100$). And thirdly, if for K_0 optimization a non-rectangular window function is used a good spectral estimate can be received only by using overlap intervals of observance that increases even more the quantity of the separate selections. All these factors bring to the necessity of coverage a very big N x F time interval for achievement founded SDP estimate of discrete process. Even if on the set relatively short F time interval the investigated process of the currency rates changes turns out stationary, then on significantly more lasting time interval N x F it will be non-stationary with high probability at big N value. In the result SDP estimate received by means of periodogram will be unfounded. In my point of view the only way out is to use parametric methods of spectral analysis that are able to get founded SDP estimate on relatively short discrete time selection with the process either stationary or that can be done stationary by removing linear trend, for example, with the help of the least squares. Among all variety of parametric methods of spectral estimation the method of spectral entropy represented for the first time by John Burg at the 37 session of the Society of exploring geophysics (Oklahoma –City) in 1937 deserves perhaps the most of attention. His fundamental report " Maximum Entropy Spectral Analysis" [5] literally shook the fundamentals of the classical spectral estimate.

Algorithm for Calculating Spectral Density of EUR/USD Currency Rate Variations on Method of Maximum Entropy

The main idea of the maximum entropy method (MME) consists of choosing such spectrum that corresponds the most random (the least predictable) time series with correlation function being coincided with set sequence of estimated values. This condition is equivalent to prediction the view of correlation function of observed time series by means of maximizing entropy of the process in theoretical and informative meaning. Exactly therefore the analysis with MME provides a significant increment of resolving capacity of S spectral estimation. Spectral estimate of the power with ME method has the same analytical form as SDP estimate received with the aid of autoregressive (AR) model of p order with entry white noise e(n). For calculation the SDP rate for the EUR/USD currency pair an autoregressive model was used of p=150 order, represented by:

$$S_{AP}(e^{j\omega}) = \left| \frac{b_0}{1 + \alpha_1 e^{-j\omega} + \alpha e^{-j2\omega} + \ldots + \alpha e^{-jp\omega}} \right|^2 \quad (1)$$

Identification of p+1 parameter a_1, a_p, b_0 of AR-pattern was realized by solution p+1 Yula-Walker's equations that in matrix form are written down as:

$$\begin{bmatrix} r_x(0) & r_x(-1) & \ldots & r_x(-p) \\ r_x(1) & r_x(0) & \ldots & r_x(-p+1) \\ \vdots & \vdots & \ddots & \vdots \\ r_x(p) & r_x(p-1) & \ldots & r_x(0) \end{bmatrix} \begin{bmatrix} 1 \\ \alpha_1 \\ \vdots \\ \alpha_p \end{bmatrix} = \begin{bmatrix} b_0^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2)$$

where $r_x(i–j)$, 1<=i<=p+1, 1<=j<=p+1, are autocorrelation coefficients serving as elements of auto regression correlation matrix $a_{1,}$ with dimension (p+1)x(p+1), and auto regression parameters form a (p+1) - dimensional vector a, with the first co-ordinate equal to 1, that is:

$$\vec{\alpha} = [1, \alpha_1, \alpha_2, \ldots, \alpha_p]^T. \quad (3)$$

System solution (2) was made with Levinson – Durbin algorithm that represents not only effective from calculating point of view procedure for defining the parameters of AR model p, but provides with effective way for definition the order of AR model that turns out equal to 150 for the EUR/USD currency rate.

Discussion on Received Results

Iterative procedure of Levinson – Durbin showed a very good convergence that proves the fact that EUR/USD rate variation is time series of autoregressive type and is generated by the next recursive ratio:

$$\hat{x}(n) = b_0 \varepsilon(n) - \sum_{k=1}^{p} \alpha_k x(n-k), \quad (4)$$

where {f(n)} – is a normalized white noise, and a normalizing coefficient b_0 is chosen so that the first vector component should be equal to 1. It is a very important collateral conclusion made in result of spectral analysis from which it follows that for the time series of EUR/USD rate variations filter construction is possible with one step in advance. Indeed, rewriting the formula (4) as:

$$\hat{x}(n) + \sum_{k=1}^{p} \alpha_k x(n-k) = b_0 \varepsilon(n), \quad (5)$$

we can estimate values x(n) by formula (5), using: a) p of known values x(n-k), b) regression parameters a_k(that are often named reflectivity factors (coefficients of mapping), c) random value e(n) from

Table 1. Characteristic and Peaks Classification Discovered in the Spectrum of Rate Variations for EUR/USD Currency Pair

Cycle Nr.	Period, Trading days	Period, Weeks	Period, Months	Amplitude, Standard units	Cycle Type
1	107.17	21.43	5.36	48.98	Basic
2	55.12	11.02	2.76	27.18	½ basic
3	35.27	7.05	1.76	10.97	1/3basic
4	28.51	5.70	1.43	11.33	28days "euro"
5	24.01	4.80	1.20	7.25	1/4basic
6	20.20	4.04	1.01	7.86	trading
7	17.56	3.51	0.87	7.35	1/6basic
8	15.39	3.08	0.76	3.61	3 weeks
9	13.52	2.70	0.68	6.20	½"euro"
10	11.43	2.29	0.57	4.23	"Alfa"
11	10.56	2.11	0.53	5.37	"Beta"
12	9.22	1.84	-	2.88	-
13	8.57	1.71	-	2.89	1/12 basic
14	7.94	1.59	-	3.09	¼"euro" triplet
15	7.45	1.49	-	2.42	¼"euro" triplet
16	7.04	1.41	-	3.45	¼"euro" triplet
17	6.36	1.27	-	2.13	6 days duplet
18	6.04	1.21	-	2.59	6 days duplet
19	5.71	1.14	-	2.12	5 days multiplet
20	5.53	1.11	-	1.51	5 days multiplet
21	5.26	1.05	-	2.28	5 days multiplet
22	5.04	1.01	-	2.08	5 days multiplet

generator of random white noise and d) coefficient b_0, and squared of $|b_0|^2$ can be considered as "prediction mistake" of linear filter. It is reasonable that here is given only the scheme solution for a very perspective task of predicting with one step in advance. Its complete solution is theoretically possible but with great efforts including that however can be justified. Let's pass to the statement of main results. The main result of the work is spectral density estimate of the power of EUR/USD currency rate. Chart of dependence

$$S_{AP} = \sqrt{S_{AP}(T)} \qquad (6)$$

is shown in Fig. 1, where T – is variations period reflected in trading days. Furthermore, speaking about the period we will mean just trading and not calendar days. Usually these charts depict function S=S(f), where f – is a normalized frequency. But in my opinion the chosen form of representing the spectrum is more convenient for information perception. In upper part of Fig.1 SDP dependence is shown in the range of periods lasting from 4 till 40 days. In the lower one – from 20 till 160 days. On datum line in Fig.1 standard units are put with dimension equal to: (EUR/USD)/(root of Hz). Spectral resolution of Sf SDP received estimate possesses characteristics quite sufficient for the digital filters adaptation. Besides, shown in Fig.1 EUR/USD spectrum can be used by technical analysts for choosing the periods of moving "averages". Many publications are dedicated to the problem of order selection the moving "averages". However it is worth mentioning that optimal selection of parameters of moving "averages" for EUR/USD cannot simply exist, as it was indicated above that EUR/USD currency rate deviations represent historical series of auto regression type and not of moving averaging-out. Full list of spectral peaks discovered in the spectrum of EUR/USD currency rate variations, and their characteristics (period and amplitude) are given in Table 1. 22 spectral components were identified, and not all of them are of the same value. Let us make their classification

At first let's try to distinguish harmonic components (harmonics), that in compliance with variation and nominal foundations of the theory of recurrence must be at any financial and commodity stock exchanges [6]. These harmonics should have periods close to 20 weeks, 40, 20, 10 and 5 days. Indeed, such harmonics were discovered in the spectrum in Fig.1. They have periods 21.43 weeks, 20.2, 10.55 and 5.04. The theory of recurrence was saved. However 40-days period failed to be identified. The 35.27-days period turned out to be closest to it. Spectral analysis showed that the cycle with the period 107.17 days or 21.43 weeks is the primary one on the EUR/USD market. Also multiple to basic cycle of harmonics with periods 55.12, 35.27, 24.01, 17.56 and 8.57, with repetition coefficients 2, 3, 6 and 12 respectively were found in the spectrum of EUR/USD in full compliance with the principle of harmony. Appearance of odd harmonics in the spectrum can be explained by the strong non-linearity of the trend. In the spectrum of market EUR/USD rate variations fairly well known trading cycle with period 20.02 days (4 weeks) occupies a deserved place. A fortnight period multiple (divisible) to trading cycle is well distinguished in the spectrum, and that can be splitted in to so-called alfa (alfa = 11.43 days) and beta (beta = 11.43 days) cycles. For the first time the terms "basic", "trading", "alfa", "beta" for the cycles description were introduced by W. Brassier. In the upper part in Fig.1 you can find weakly marked spectral peak with period of (15.39days) close to 3 weeks. Special attention should be paid to the spectral peak with the period of 28.51 days that in the upper part in Fig.1 has maximal amplitude. In the trading days quantity (but not in calendar continuity) its period coincides with the period of so called lunar cycle specified by the Moon phases. Later on we will name this 28-days cycle as "euro"- cycle, because obviously it is typical for the EUR/USD spectrum. The following fact serves to confirm this supposition that its multiple harmonic with the period of 13.5 days forms in the spectrum the sharpest (the most pointed) peak located between two deep "downfalls".

In other words this spectral line has a very high ratio signal/noise. Seven days "euro" triplet is a result of splitting 7-days harmonic, multiple to 28-days "euro" cycle. Weak spectral peak with the period of 9.22 days is rather difficult to be classified exactly. Most likely this is odd (coefficient 3) harmonic of the 28-days "euro" cycle (28.51/3=9.5 R 9.22). However it is quite possible that this is a result of the 10 days harmonic splitting. Spectral analysis of weekly exchange charts of the EUR/USD currency rate is necessary for distinguishing more lasting seasonal cycles with periods in one year and long-term cycles with periods in 2 years and more. Perhaps, this analysis will be made in future with the purposes of long-term rate forecast but within the framework of the task the trading algorithm development on the base of AT&CF method this is not required.

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